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ONTARIO

DEPARTMENT OF EDUCATION

MATHEMATICS

Intermediate Division
Grade 8

For introduction on an optional basis in Grade 8 for students who have completed Curriculum I.12A Mathematics

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MATHEMATICS

Grade 8

Course Options

1. Where Curriculum I.12A (Mathematics) has been followed in Grade 7, Curriculum I.12B should be followed in Grade 8.

2. Where Curriculum I.1 (1951) has been followed in Grade 7, either of the following options may be chosen in Grade 8:

   (a) Curriculum I.1 (1951), pages 151 to 157

   (b) Curriculum I.12A, and the following additional topics

      (i) The circle (details from Curriculum I.1, page 156, or from Unit 6 of I.12B)

      (ii) The metric system (details from Curriculum I.1, page 156, or from Unit 6, part 4 of I.12B)

      (iii) Further practical problems applying per cent or ratio (details from Curriculum I.1, Page 155, or from part 2 of Unit 5 of I.12B)

Notes on Option 2 (b)

(1) This option may be chosen only in the year when new approach mathematics is being introduced into a school.

(2) Having covered the Grade 7 Mathematics Course in Curriculum I.1 (1951), pupils will have studied some aspects of fraction and decimal notation, and of measurement as outlined in Curriculum I.12A. As a result, there will be time in Grade 8 to cover all of Curriculum I.12A and the three additional topics noted in section 2 (b) above.

(3) It should be noted that those who have followed this option will, at the end of their Grade 8 year, have covered all of the Grade 7 and Grade 8 topics in Curriculum I.1.

COURSE OUTLINE

Unit 1: Extension of the Language and Application of Sets (2 weeks)

In Grade 7, pupils were introduced to the concept of the set. In that grade, they used the concept in the study of the whole numbers, of the fractional numbers, and in the geometrical figures treated as sets of points. The only symbols actually required in Grade 7 were the braces for enclosing a set and the three-dot symbol meaning *and so on.* This unit now extends the ideas to include some of the other symbols and introduces the language and methods used in combining sets.

Note: Part 1 below is to be interpreted as a recall of the symbols and ideas developed in Grade 7. Since Part 2 is to serve as an informal introduction to further use of sets, formality is to be avoided, and the use of symbols for union and intersection is to be considered optional.

1. Review of the use of braces for writing sets of places, people, numbers

   Review of the writing of subsets of the set of whole numbers, e.g., those less than 8, greater than 5, equal to or greater than 4

   Review of the symbols $>$, $<$, $\neq$, $\geqq$, $\leqq$

   Review of the three-dot convention for *and so on*

   Practice in converting sets described by words to sets described by symbols and vice versa

2. Development and practice with the symbol for *is a member of*

   Some practice with the cases where the set is the empty (or null) set

   Practice in writing all of the subsets of given sets

   Development of the intersection set of two or three given sets

   Development of the union set of two or three given sets

   The use of Venn Diagrams to picture intersection and union sets

Unit 2: A Review and Extension of the Properties and Operations in the Set of Whole Numbers (5 to 6 weeks)

Section A of the Grade 7 outline developed the properties of the set of whole numbers (i.e., the natural numbers and zero), the use of number phrases and number sentences involving whole numbers, factoring and the use of exponents with whole numbers, and the use of numeration system to bases other than ten. Unit 2 in Grade 8 reviews and extends this work with whole numbers.

Notes:

(1) In this unit and in the following units, solution of problems is an important feature and must have an important place at each step of the development, even though not specifically written into the course outline.

(2) For various reasons, the noun *exponent* is favoured over its accepted synonym *index*. The Grade 7 course outline allowed the use of either term in deference to the teachers' past experience that, in most cases, will have been limited to the use of *index*. Either term may still be used but every encouragement is given to develop the use of exponent. Accordingly, only the term *exponent* is used in this outline.

1. Review of the picturing (graphing) on a number line of subsets of the set of whole numbers

2. Review of the solution by inspection of simple number sentences involving equalities and inequalities in the set of whole numbers

   Graphing of solution sets on a number line

3. Review of the order of operations for simplifying number phrases or expressions

   Practice in deriving equivalent expressions for given open number phrases or expressions, with the replacement set being restricted to the set of whole numbers

   Introduction to the use of *term*

4. Review of the use of the exponent as a short form of indicating the number of times the same factor appears in a product

Review of the calculations of given powers

Development from problems of the relative priority of powers in the order of operations and practice with simple open number phrases involving powers

Area and volume significance of *the square of* and *the cube of* when using the exponents 2 and 3

Calculation of the squares of the whole numbers 1 to 15 inclusive

Introduction to "square root" and the recognition of the square roots of 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, 196, 225

(The formal square root algorithm is not to be taught)

(Supplementary) Study of various special number patterns leading to the discovery of those whole numbers that form the Pythagorean groups, e.g., 3, 4, and 5, because $3^2 + 4^2 = 5^2$

5. Review of place value in whole numbers and of writing whole numbers in expanded notation, e.g.,

   $2546 = 2 \times 10^3 + 5 \times 10^2 + 4 \times 10 + 6 \times 1$

   Review of numeration with other bases and extension to include base two

   Changing to and from base ten

   Some practice in adding and subtracting in other bases

   Reference to the use of base 2 with electronic computers

   (Supplementary) Multiplication with other bases

6. Review of the properties of the set of whole numbers

   The formal statement of the commutative property, the associative property, the distributive property

   Practice with open number phrases and sentences applying these properties, stressing particularly the use of the distributive property

   Review of Section A, Unit 1, item 6 of the Grade 7 course and extension of the concept to include the term *closure*

7. Review of the operations of addition, subtraction, multiplication, and division, stressing the inverse operations

Review of the operations with zero and unity and of the need of excluding division by zero

Extension to include the term *identity element*

8. Practice in writing number sentences to establish clearly that number sentences are translations of the operations and the sequences indicated in word problems

9. Practice in problem-solving using number sentences

Unit 3: A Review of the Properties of the Set of Fractional Numbers (3 to 4 weeks)

Units 1 and 2 of Section B of the Grade 7 outline developed the operations with fractional numbers and the conversions from one notation to another. It was indicated that the operations of addition and multiplication were commutative and associative and that multiplication was distributive over addition. This unit in Grade 8 is developed to review these operations and properties and to extend the work with the fractional numbers to include the use of exponents, the evaluation of open number phrases, and determination of solution sets of simple cases where fractional numbers are involved.

1. Definition of a fractional number $\frac{a}{b}$ such that either of $b \times \frac{a}{b}$ or $\frac{a}{b} \times b$ is equal to a

2. Review of addition, subtraction, multiplication, and division in decimal notation

   Review of addition, subtraction, multiplication, and division in fraction notation, including some practice with horizontal arrangements in addition and subtraction

   Problems involving these operations

3. Review of the operations to show that
   (i) addition and multiplication are commutative and associative
   (ii) multiplication is distributive over addition
   (iii) addition and subtraction are inverse operations

(iv) multiplication and division are inverse operations

(v) the set is closed with respect to addition, multiplication, and division (division by zero excluded)

(vi) zero is the identity element in addition

(vii) unity is the identity element in multiplication

4. Inductive development with equivalent fractions to establish the general statement of the Principle of Equivalent Fractions

5. Simplifying of open number phrases or expressions with the replacement set being the set of fractional numbers

   The determination by inspection of the solution set of equations with the replacement set being the set of fractional numbers (only cases simple enough to be solved by inspection)

   The graphing of solution sets on a number line

6. Simplifying powers where the base is a fraction

   Simplifying open number phrases which involve exponents and in which the placeholder is a fraction

7. Square roots of numbers in fraction notation where both numerator and denominator are squares of whole numbers, e.g., $\frac{4}{9}$, $\frac{16}{49}$

   Square roots of a few of the whole numbers 1 to 225 determined to one decimal place by inspection and trial (using judgment according to proximity to 1, 4, 9, 16, 25 . . . 169, 196, 225, and multiplying out to check judgment)

   Note: No formal square root algorithm to be taught

Unit 4: A Study of the Integers (9 to 10 weeks)

Throughout Grade 7 and the previous units of Grade 8, the set of whole numbers and the set of fractional numbers have been studied. For each set, the operations of addition, subtraction, multiplication, and division have been studied,

and the commutative, associative, and distributive properties have been recognized as being characteristics of each set.

Solution sets have been determined for given number sentences using either the set of whole numbers or the set of fractional numbers as the replacement set, and these solution sets have been graphed upon number lines. It is to be noted, however, that it is not always possible to obtain a solution set, e.g., in $n + 5 = 3$.

Furthermore, it has been discovered that the set of whole numbers is closed with respect to addition and multiplication, and that the set of fractional numbers is closed with respect to addition, multiplication, and division. At this stage, we have no set that is closed with respect to subtraction.

In this unit, a new set of numbers is introduced that is closed with respect to subtraction as well as with respect to addition and multiplication, and hence this set extends the range of problems that may be solved. This new type of number involves the two ideas of magnitude and sense (or direction). Because signs are used to indicate the direction or sense, these are sometimes spoken of as *signed numbers*. Signed numbers eventually will include signed fractional numbers and will be closed with respect to division also. In Grade 8, however, the study will be restricted to integers in order to simplify the development of the operations where signs of quality are involved.

1. Establishment of the need of another set of numbers (see points developed in the preamble)

2. Introduction of the use of directed line segments (using an arrow pointing left or right to indicate direction)

   Considerable practice in calculating the combined result of two trips in the same or opposite direction (later extension to more than two trips)

   Discovery of the property of opposites

3. References to the common occurrence of opposite direction or sense in daily life, e.g., gain and loss, up and down, hot and cold, charge and discharge

   Examples of combining numbers of *opposite direction or sense*

Use of *positive and negative numbers* with the sense designated by signs placed before the numerals in the "raised" position

Considerable practice in combining these using the directed distance concept (no rules)

Notes:

(1) The raised signs should be maintained all through Grade 8.

(2) These signs are to be read as *positive* or *negative*, not as plus or minus.

4. The use of positive and negative numbers also to indicate position on a line (note that in parts 2 and 3 above, positive and negative numbers were used to indicate directed distances only)

   Graphing of the integers on a number line

   Reference to negative temperature *readings* as a familiar use of negatives to indicate *position* on a number line

Note: It should be recognized by the teacher that, when signs are used to indicate *changes* in temperature, the positive and negative numbers are then being used to show *direction*. It is important that the two-fold use of positive and negative numbers be recognized. In actual practice the distinction between the two soon disappears and one finds that he is switching back and forth between the uses according to which suits best, e.g., in addition and subtraction, directed distance is the more profitable idea, while in the interpretation of solution sets or in study of relative position, the position idea is more profitable.

5. Addition of integers

   (Note that utilization of the direction idea should make unnecessary any rules or any separation into the integers with like or unlike signs)

   Practice in addition using horizontal arrangement of the terms

6. Subtraction of integers

   Considerable practice, including some with vertical arrangement of the terms

Note: Some practice in subtraction of integers without the use of a rule is to be encouraged. When a rule is introduced, the one to be used is "to subtract an integer, add its opposite."

7. Multiplication of integers

   Inductive development of the definition of signs for multiplication of *two* integers

   Considerable practice in multiplying two integers

   Extension to multiplication of more than two integers treated as a repeated two-factor process with no formal attempt made to establish a rule for more than two factors

8. Division of integers treated as the inverse of multiplication, as in $^{-}6 \times n = {}^{-}24$

9. Illustration that the commutative, associative, and distributive properties are characteristics of these operations as we have developed them

   Practice using open number phrases and sentences applying these properties with particular stress on the use of the distributive property

   Recognition that this set is closed with respect to addition, multiplication, and subtraction

10. Simplifying open number phrases where the replacement set is the set of integers

11. Discovery by inspection of the solution sets of number sentences involving equalities and simple inequalities where the solution may include integers

    The graphing of these on number lines

12. Practice in setting up number sentences for word problems where the solution may include integers

13. (Supplementary)

    Review of expanded notation, e.g., 4356.317 written as

    $$4 \times 10^3 + 3 \times 10^2 + 5 \times 10 + 6 \times 1 + 3 \times \frac{1}{10} + 1 \times \frac{1}{10^2} + 7 \times \frac{1}{10^3}$$

Use of the idea of the integers and the pattern of decreasing value of the exponents to establish the equivalent form

$$4 \times 10^{+3} + 3 \times 10^{+2} + 5 \times 10^{+1} + 6 \times 10^{0}$$
$$+ 3 \times 10^{-1} + 1 \times 10^{-2} + 7 \times 10^{-3}$$

Intuitive determination of a meaning for each of $10^{0}, 10^{-1}, 10^{-2}, 10^{-3}$ from these two equivalent forms

Previous experience with scientific (or standard) notation extended to numbers less than unity

Use of scientific (or standard) notation to express very large and very small numbers

Unit 5: Ratio (4 to 5 weeks)

In Grade 7, Unit 3 of Section B introduced the use of ratio for comparisons and developed the test for equivalent ratios and the concept of per cent as a special two-term ratio of which the second term is 100. These concepts were then used to solve problems in discount, interest, and commission by the use of ratio. This unit develops the topic of ratio to include rates and carries the application to further types of problems.

Note: It is intended that problems in this unit dealing with such topics as taxation and insurance are to be direct applications of the use of ratio. It is not intended that mention of these topics implies an exhaustive treatment of the social implications and more difficult problems sometimes associated with them.

1. Review of two-term ratios and determining missing terms by inspection

   Review of the test for equivalent ratios

   Application to conversions and to problems in discount, insurance, and commission

   (Supplementary) Ratios of more than two terms

2. Introduction to *rates* as in rate of interest per annum, miles per hour, mills per dollar of assessment, revolutions per second, dollar cost per article

Use of rates as two-term ratios and applications to problems of travel, growth, and cost, using the test for equivalent ratios

Review of annual interest and extension to terms of other than one year

3. Review of the Grade 7 interpretation of graphs used for ordinary comparisons

   Extension to line graphs to show rates of increase or decrease

   Use of a line-graph to represent costs when given a certain cost per unit (e.g., gallon) to show the significance of rate when indicated graphically

Unit 6: Geometry and Measurement (6 to 7 weeks)

Section C of the Grade 7 Course outlined the geometrical concept of certain simple straight line figures and the measurement of length, area, and volume of some of the simple figures.

This unit extends the geometrical concepts to include the circle and some of the simpler solids and the measurements related to some of these. In addition, pupils are given the opportunity of extended use of construction to allow discovery of geometrical and measure relationships.

1. (a) A brief review of the triangle as one of the set of simple closed figures

   (b) Practice in the construction of triangles when given the measures of three sides, of one side and two adjacent angles, or of two sides and a contained angle

   (c) Discovery of the fact that the particular combinations of three measurements given in (b) are sufficient to determine one (and only one) triangle, whereas fewer than three measurements or combinations of three measurements other than those in (b) create difficulty in determining one (and only one) triangle

2. (a) Discovery of the ruler-and-compasses method of constructing
   (i) the bisector of a line segment
   (ii) the bisector of an angle
   (iii) a line perpendicular to a line at a point in the line
   (iv) a line perpendicular to a line from a point outside the line
   (v) a line parallel to a given line
   (vi) an angle equal to a given angle

   (b) Practice in using these methods

3. Inductive discovery of geometrical relations in triangles and other geometrical figures, for example,
   (i) the angles opposite the equal sides of an isosceles triangle are equal
   (ii) the angle sum of a triangle is 180°
   (iii) the area of the square on the hypotenuse of a right-angled triangle is equal to the sum of the areas of the squares on the other two sides
   (iv) the alternate angles in parallel line configurations are equal
   (v) the corresponding angles in parallel line configurations are equal

Note: In the discovery of these geometrical relations, pupils should be made aware that this is an example of one type of reasoning. They should be led to see that the study of examples, recognition of a possible relation, conjecture, and testing of the conjecture is a method of reasoning leading to a theory, and that this is a type of reasoning used extensively in science.

4. Development of the concept of a circle as a set of points each of which is equidistant from a given fixed point

   Special points and line segments associated with the circle (centre, diameter, radius)

   Use of the method indicated in part 3 above to discover the relation between diameter and radius, and between perimeter and diameter

Discovery of the relation between the radius and the area of the (interior) region of a circle

Problems involving the perimeter and area of the interior region of a circle

5. Review of the measurements (perimeter, area, and volume) associated with the rectangular solid

Classification of the rectangular solid as a rectangular prism and extension of the prism concept to include the triangular prism and other right prisms (use of models)

Discussion to discover the characteristic common to all of these prisms (that the edges are parallel)

Diagrams of prisms to show the three-dimensional aspect

Re-examination of the volume of the rectangular solid to establish the measure of its volume as the product of the measures of its base area and its height

Extension of this concept of volume to establish the volume of the triangular prism

Extension of this concept of volume to establish that of the cylinder

Problems involving lengths, areas, and volumes of the triangular and rectangular prisms and the cylinder

Note: At this stage problems involving the use of formulas should stress those in which the formula may be used directly without change of the subject. It should be recognized that the best approach to the change of even the simple formula, $A = b\,w$, to $b = \frac{A}{w}$ involves the use of the number properties, and that topic has been deliberately omitted from the work on equations in Grade 7 and 8. This restriction will, however, not prevent an intuitive solution of a problem in which area and width are given, or the intuitive solution (without manipulation) of a number sentence such as $8 \times w = 36$ resulting from such a problem.

6. The metric system of units of measure for lengths, areas, and volumes

Practice in measurement using these units

Practice in conversions within the metric system

Problems involving the use of the metric system

Note: The primary objective of this study of the metric system is a development of an appreciation of the units and of their relationships within the system. For this reason, the conversions to and from English equivalents have been deliberately omitted.

Unit 7: An Introduction to some of the Language used in the Presentation and Interpretation of Data (3 to 4 weeks)

More and more in our modern world, the science of communication of ideas is depending upon the support of collected data. Hence it becomes more and more important that the consumer be given some opportunity to acquire the knowledge needed to interpret data as presented in tabulated or graphical form. Unit 3 of Section A of the Grade 7 outline introduced an interpretation of picto-graphs and bar-graphs to illustrate comparisons. Unit 5 of the Grade 8 outline introduced the line-graph to illustrate rate. These are now reviewed and extended to include the circle-graph. In addition, terms used in representing central tendencies, namely, average or arithmetic mean, median, and mode are introduced.

Closely related to the problems of interpreting data is the significance of the term *probability* now entering into people's lives either directly or through the decisions of finance, commerce, and industry based on the mathematics of probability. This unit introduces the mathematical concept of probability by the use of simple trials where the number of cases may be counted. In the last part, reference is made to the use of this same ratio in finance, commerce, and industry where the favourable and total cases cannot be counted but must be approximated from collected data.

1. Review of the interpretation of picto-graphs and bar-graphs

   Review of the line-graph to show rate, and extension to include other uses

   Interpretation of the circle-graph

   Use of the histogram to picture grouped data

2. Calculations of central tendencies

   (a) The arithmetic mean

   (b) The median

   (c) The mode

Note: Some writers treat *average* as a synonym for *arithmetic mean,* and others treat it as a synonym for *measures of central tendency,* (and hence including mean, median, mode, and weighted mean). Since the former has been the accepted practice in Ontario schools, it is suggested that this interpretation be continued for the present.

*3. (a) Development of the concept of probability as the ratio of the number of "favourable" cases to the total number of cases

   (b) Calculation of probabilities in simple cases where the favourable and total cases can be easily discovered, e.g.,

      (i) the probability of obtaining a head on flipping one coin

      (ii) the probability of getting a four on rolling a die

      (iii) the probability of getting a total of more than three on rolling a die

      (iv) the probability of getting all 'heads' on flipping two coins, three coins, or four coins

      (v) the probability of getting a total of more than seven on rolling two dice

      (vi) the probability of picking two red marbles from a collection of three reds, one blue, and one green

*4. Collecting and tabulating the results of many separate trials with coins, dice, and marbles to test the ratio $\frac{\textit{favourable cases}}{\textit{total cases}}$ obtained by experiment with that obtained by calculation

Note: The ordinary result is that the greater the number of trials, the closer the ratio obtained by experiment will be to the probability calculated by mathematics. Devices should, therefore, be used to get the results of a great number of trials, e.g., each pupil in a class of 35 does ten trials and records results, and then the results of the 350 trials are combined.

*5. Discussion of cases in commerce and industry where the obtaining of this ratio is important, e.g., what should be the ratio of men's shoes of size 9½ to the total number of men's shoes manufactured?

Discussion of means used to obtain such information (use of data collected from a representative sample of population)

Recognition of the similarity of this means to the experiments used in part 4 above

***Note:** During the first years of use of this course, and until further notice, the last three parts of Unit 7 (parts 3, 4, 5) may be considered as supplementary material.